THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH2060B Mathematical Analysis II (Spring 2017)
Tutorial 7

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1. (a) State the change of variable formula.

   (b) Let φ be strictly positive and continuously differentiable on $\mathbb{R}$, and $x > 0$. Using change of variable formula, show that

   $$\int_0^x \frac{\varphi'(t)}{\varphi(t)} dt = \ln \varphi(x) - \ln f(0)$$

   (Q: Why is left hand side Riemann integrable?)

   (c) State the integration by parts formula.

   (d) (**Seems out of scope at present**) Using integration by parts, show that the following limit of integral converges:

   $$\lim_{R\to\infty} \int_1^R \frac{\sin x}{x} dx$$

   Remember that we should first show that for each $R > 0$, the function $\frac{\sin x}{x}$ is Riemann integrable on $[1, R]$.

   (e) Let $f : [a, b] \to \mathbb{R}$ be smooth, $c \in (a, b)$. Prove the following Taylor's theorem with a precise form for the remainder:

   $$f(x) = \sum_{k=0}^{n} \frac{f^{(k)}(c)}{k!}(x - c)^k + R_n(x),$$

   where

   $$R_n(x) = \frac{1}{n!} \int_c^x f^{(n+1)}(t)(x - t)^n \, dt$$